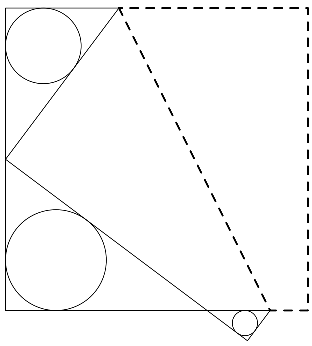
# VIII Geometrical Olympiad in honour of I.F.Sharygin
# Final round. First day. 8 form

*Ratmino, 2012, July 31*

1. Let $M$ be the midpoint of the base $AC$ of an acute-angled isosceles triangle $ABC$. Let $N$ be the reflection of $M$ in $BC$. The line parallel to $AC$ and passing through $N$ meets $AB$ at point $K$. Determine the value of $\angle AKC$.

2. In a triangle $ABC$ the bisectors $BB'$ and $CC'$ are drawn. After that, the whole picture except the points $A$, $B'$, and $C'$ is erased. Restore the triangle using a compass and a ruler.

3. A paper square was bent by a line in such way that one vertex came to a side not containing this vertex. Three circles are inscribed into three obtained triangles (see Figure). Prove that one of their radii is equal to the sum of the two remaining ones.



4. Let $ABC$ be an isosceles triangle with $\angle B = 120°$. Points $P$ and $Q$ are chosen on the prolongations of segments $AB$ and $CB$ beyond point $B$ so that the lines $AQ$ and $CP$ are perpendicular to each other. Prove that $\angle PQB = 2\angle PCQ$.

# VIII Geometrical Olympiad in honour of I.F.Sharygin
## Final round. First day. 9 form.

*Ratmino, 2012, July 31*

1. The altitudes $AA_1$ and $BB_1$ of an acute-angled triangle $ABC$ meet at point $O$. Let $A_1A_2$ and $B_1B_2$ be the altitudes of triangles $OBA_1$ and $OAB_1$ respectively. Prove that $A_2B_2$ is parallel to $AB$.

2. Three parallel lines passing through the vertices $A$, $B$, and $C$ of triangle $ABC$ meet its circumcircle again at points $A_1$, $B_1$, and $C_1$ respectively. Points $A_2$, $B_2$, and $C_2$ are the reflections of points $A_1$, $B_1$, and $C_1$ in $BC$, $CA$, and $AB$ respectively. Prove that the lines $AA_2$, $BB_2$, $CC_2$ are concurrent.

3. In triangle $ABC$, the bisector $CL$ was drawn. The incircles of triangles $CAL$ and $CBL$ touch $AB$ at points $M$ and $N$ respectively. Points $M$ and $N$ are marked on the picture, and then the whole picture except the points $A$, $L$, $M$, and $N$ is erased. Restore the triangle using a compass and a ruler.

4. Determine all integer $n > 3$ for which a regular $n$-gon can be divided into equal triangles by several (possibly intersecting) diagonals.

# VIII Geometrical Olympiad in honour of I.F.Sharygin
# Final round. First day. 10 form.

*Ratmino, 2012, July 31*

1. Determine all integer $n$ such that a surface of an $n \times n \times n$ grid cube can be pasted in one layer by paper $1 \times 2$ rectangles so that each rectangle has exactly five neighbors (by a line segment).

2. We say that a point inside a triangle is *good* if the lengths of the cevians passing through this point are inversely proportional to the respective side lengths. Find all the triangles for which the number of good points is maximal.

3. Let $M$ and $I$ be the centroid and the incenter of a scalene triangle $ABC$, and let $r$ be its inradius. Prove that $MI = r/3$ if and only if $MI$ is perpendicular to one of the sides of the triangle.

4. Consider a square. Find the locus of midpoints of the hypothenuses of right-angled triangles with the vertices lying on three different sides of the square and not coinciding with its vertices.